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Assessing The Performance of Forecasting Equity Returns Using Various Estimators of Beta

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Abstract

In this study, we test the performance of various beta estimators in their ability to forecast future returns. Using Apple. Inc. (AAPL), we estimate beta based on four market indices across three different time intervals. We then use the Fama-French three-factor model to estimate the returns for the following year. Finally, we use more traditional measures of performance, MSE, and MAD, and a less common Pitman Closeness to evaluate the beta-based estimators' performance.

Introduction

Assessing the performance of asset pricing models goes back decades, and as Ang et al. (2020) explain, there seems to be no accurate, consistent assessment method. Stock return volatility has been measured and studied numerous times. For example, Chow and Lahtinen (2019) studied realized volatility using high-frequency data and concluded that a shorter time interval provides the best measure of realized volatility. This finding contradicts Anderson et al. (2001) and Liu et al. (2015), who suggest a five-minute interval as the superior interval for estimating realized volatility.

The volatility of returns has been measured by both the variance of the changes in returns within a time interval and the beta of a return as a regression coefficient related to a measure of the overall market, such as the S&P 500. Lahtinen, Lawrey, and Hunsader (2018) note that various methods exist to estimate the beta of a return related to the market. For example, "on March 14, 2017, Yahoo Finance, Google Finance, and Nasdaq.com reported Apple Inc.'s beta to be 1.45, 1.25, and 0.72, respectively." The reasoning for the difference in beta estimation is simple; there is no one correct and agreed-upon method to estimate beta. In their study, they used three return windows (daily, weekly, and monthly), three market proxies (CRSP Equal, S&P 500, CRSP Value), and three observations (corresponding to the return window), which led to 27 beta estimates.

Estimating future returns is of interest to most investors as it provides a way of assessing future outcomes for investment planning purposes. The capital asset pricing model (CAPM) (see Sharpe, 1964, Lintner, 1965, and Mossin, 1966) brought about a regression-based model for estimating the prices of securities, which in turn could be utilized as a forecasting method. Fama and French (1993 and 1996) expanded the CAPM model to a three-factor model, adding factors or variables of the difference in returns between small and large companies (SMB) and the difference in returns between high and low book-to-market ratio companies (HML). Others have expanded this further into four and five-factor models (Carhart, 1997 and Fama & French, 2015). Estimating beta to forecast returns have been studied in not only US markets but also in markets abroad (Ali & Badhani, 2021, Hollstein et al, 2019).

Pitman (1937) introduced a method for comparing estimators of a population parameter by determining the better estimator as the estimator who is closer to the actual value at a greater probability. Simply put, the closer estimator is the one that provides an estimate closer to the true value more often. While most who use this approach have focused on applications within the realm of pure statistics, some have ventured to apply this technique in other fields. For example, Chow, Lahtinen, & Pennywell (2018) used Pitman to evaluate estimators of volatility using range-based estimators. Javine, Pennywell, and Chow (2014) used Pitman to compare estimates of return prices using Fama-French portfolios. Pennywell, Chow and Javine (2014) conducted a similar study when comparing various industry portfolio returns including the energy sector, looking for changes in the pre and post period to Enron's bankruptcy.

This study incorporates these methods to assess various measures of beta based on several market measures and then uses the Fama-French three-factor model as a proven method of estimating future returns. We then use more standard Mean Squared Error (MSE), Mean Absolute Deviation (MAD), and Pitman Closeness (PC) to evaluate the performance of the betas in estimating future returns.

Methodology

Data for this study were downloaded from the Yahoo Finance website. They consisted of the daily prices for Apple (AAPL) and the market indices Russell 2000 (RUT), NASDAQ (IXIC), Center for Research in Security Prices (CRSP), and S&P 500 (GSPC). Data for January 2016 through December 2018 were regressed to estimate beta coefficients for four models, one for each of the previously mentioned market measures. In order to remove the impact of the COVID-19 pandemic's effect on the markets, we studied the four year period prior to the pandemic in the US, which is marked mostly as March 2020 (Albulescu, 2021, and Chowdury et al, 2022).
Returns were calculated as

$$R_t = log\frac{P_t}{P_{t-1}} \qquad (1)$$

Returns were calculated for the daily closing prices. The Fama-French three-factor model (Fama & French, 1993 & 1996) was used to estimate the daily returns for 2019. Additional data, such as the factors for the Fama-French model, were downloaded from the Kenneth French website on the Dartmouth Data Library. The components of the three-factor model

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + S_i(SMB_t) + H_i(HML_t) + \varepsilon_{it} \qquad (2)$$

Where,

R_{it} = realized return on security i at time t;
R_{mt} = realized return on the market at time t. Downloaded from the Dartmouth Data Library as the value-weighted return on all NYSE, AMEX, and NASDAQ stocks minus the one-month T-bill rate;
R_{ft} = nominal risk-free rate of return at time t;
α_i = the intercept, constant term for security i;
β_i = slope coefficient for security i on the market risk factor;
ε_i = the residual excess return on portfolio i during time t;
S_i = slope coefficient for security i on SMB;
H_i = slope coefficient for security i on HML;
SMB_t = the difference in returns on small versus large firms during time t;
HML_t = the difference in return on high versus low book-to-market ratios during time t.

Using the intercept and slope coefficients calculated using the daily returns from 2016 – 2018, we estimated returns for 2019 using the three-factor model. Estimated returns were compared to actual returns, and the efficiency of the estimator using each of the slopes calculated for each market measure was assessed using the Mean Squared Error (MSE), Mean Absolute Deviation (MAD), and Pitman Closeness (PC). Pitman (1937) proposed that an appropriate measure of the efficiency of an estimator would be to determine the closest estimator to a parameter; Pitman's "closeness" is defined as follows:

If $\hat{\theta}_1$and $\hat{\theta}_2$ are consistent estimators of a parameter □□□then $\hat{\theta}_1$ is a "closer" estimator for □ than $\hat{\theta}_2$ if, for all □

$$\Pr\left(\left|\hat{\theta}_1 - \theta\right| < \left|\hat{\theta}_2 - \theta\right|\right) > {}^{1}\!/_{2} \qquad (3)$$

In this study, we shall rely on more than one measure of the efficiency of the estimator to provide as much information as possible regarding the performance of each estimator.

Results

The results of this study are summarized in Table 1. Panel A provides the results for the daily return estimations. Based on the beta coefficients related to the Russell 2000 market measure, the estimator had the lowest MSE and MAD, followed by the NASDAQ, CRSP, and S&P 500. Using the Pitman Closeness criterion (PC), the Russell 2000 based estimator was PC than each of the other estimators, with the NASDAQ PC than the CRSP and S&P 500, and the CRP PC than the S&P 500.

Panel B provides the results for the weekly return estimations. The estimator based on the beta coefficients related to the CRSP market measure had the lowest MSE and MAD, followed by the Russell 2000, the NASDAQ, and the S&P 500. Using the Pitman Closeness criterion, the CRSP-based estimator was more PC than the other market-based estimators tested. The NASDAQ was PC than the Russell 200 and the S&P 500, and the Russell 2000 based estimator was PC than that of the S&P 500.

Panel C provides the results for the monthly return estimations. The estimator based on the beta coefficients related to the S&P 500 market measure had the lowest MSE and MAD, followed by the Russell 2000, CRSP, and NASDAQ. Using the Pitman Closeness criterion, the S&P 500 based estimator was more PC than the other market based estimators. The Russell 2000 based estimator was PC than the NASDAQ and CRSP based estimators. The CRSP based estimator w]as PC than the NASDAQ based estimator.

Table 1 – Efficiency of Estimators

Panel A - Daily			Pitman			
Market Measure	MSE	MAD	RUT	IXIC	CRSP	GSPC
RUT	0.000111	0.007251		42%	30%	32%
IXIC	0.000145	0.008566	69%		60%	58%
CRSP	0.000137	0.008274	70%	40%		52%
GSPC	0.000136	0.008247	68%	42%	48%	
Panel B - Weekly			Pitman			
Market Measure	MSE	MAD	RUT	IXIC	CRSP	GSPC
RUT	0.0010740	0.0265060		98%	31%	62%
IXIC	0.0010800	0.0264590	2%		29%	10%
CRSP	0.0005820	0.0192420	69%	71%		69%
GSPC	0.0011080	0.0268110	38%	90%	31%	
Panel C - Monthly			Pitman			
Market Measure	MSE	MAD	RUT	IXIC	CRSP	GSPC
RUT	0.0001323	0.034785		8%	17%	58%
IXIC	0.002542	0.04812	92%		25%	75%
CRSP	0.004509	0.061528	83%	83%		83%
GSPC	0.0010387	0.025077	42%	25%	17%	

The results from this brief study show that using a different estimate for the beta can and will lead to a different result when using those beta estimates in predicting future returns.

Conclusions

While the data for this study only focused on one equity, as Banerjee (2020) did, it shows that when using betas based on different market measures and time periods, the best or closest estimators may come from other beta estimates. While the beta is used as a measure of an equity's relation to the overall market, using it within the context of asset pricing models leads to some concern for those making this application. As Lahtinen, Lawrey, and Hunsader (2018) noted, many methods exist to estimate beta. Which beta is used in the estimation of future returns will likely impact the estimator's performance. Those interested in assessing in this manner should be cautious and look at all the methods available for estimating beta and, in turn, estimating returns.

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